

Observing gravitational lensing effects by Sgr A* with GRAVITY

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ABSTRACT

The massive black hole at the Galactic center Sgr A* is surrounded by a cluster of stars orbiting around it. Light from these stars is bent by the gravitational field of the black hole, giving rise to several phenomena: astrometric displacement of the primary image, the creation of a secondary image that may shift the centroid of Sgr A*, magnification effects on both images. The near-to-come second generation VLTI instrument GRAVITY will perform observations in the Near Infrared of the Galactic Center at unprecedented resolution, opening the possibility of observing such effects. Here we investigate the observability limits for GRAVITY of gravitational lensing effects on the S-stars in the parameter space $[D_{\text{LS}}, \gamma, K]$, where D_{LS} is the distance between the lens and the source, γ is the alignment angle of the source, and K is the source apparent magnitude in the K -band. The easiest effect to be observed in the next years is the astrometric displacement of primary images. In particular the shift of the star S17 from its Keplerian orbit will be detected as soon as GRAVITY becomes operative. For exceptional configurations it will be possible to detect effects related to the spin of the black hole or Post-Newtonian orders in the deflection.

Subject headings: Gravitational lensing — Black hole physics — Galaxy: center — astrometry — infreared: stars — instrumentation: interferometers

1. Introduction

Understanding the nature and the physics of a black hole has always fascinated generations of scientists as well as ordinary people. Obviously, the first basic question is whether there exist black

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holes actually. Only recently the astronomers have been able to gain quite conclusive evidence to support a positive answer. For instance, it is widely accepted that the central engine of an AGN is an accreting supermassive black hole ($M_{\bullet} > 10^6 M_{\odot}$) at the center of the galaxy, responsible of the observed high-energy emission; stellar black holes ($3 M_{\odot} < M_{\bullet} < 10 M_{\odot}$) are observable in close binary systems, when matter is transferred from a companion star to the black hole; many ultra-luminous X-ray sources are finally suspected to be intermediate-mass black holes ($M_{\bullet} > 10 M_{\odot}$). However, the royal evidence of the black-hole existence resides in the innermost part of our Galaxy, and comes from decennial near-infrared (NIR) observations of the orbital motion of more than two dozen stars (the so-called *S-stars*) around the well-known and motionless compact radio source, Sgr A*, which is indeed a massive black hole of about $4.3 \times 10^6 M_{\odot}$ (Eckart et al. 2002; Schödel et al. 2003; Ghez et al. 2005a; Eisenhauer et al. 2005; Ghez et al. 2008; Trippe et al. 2008; Gillessen et al. 2009a; Schödel et al. 2010). In particular, more than 15-years continuous monitoring of this stellar cluster allowed to observe the whole orbit of the star S2, which completed a full revolution around Sgr A* in 2008 (Gillessen et al. 2009b). For an exhaustive review on the argument, we refer the reader to Genzel et al. (2010).

Therefore, Sgr A* represents by far the best (and closest) place for searching for post-Newtonian effects in strong-field regime, like for instance the relativistic motions of flaring jets and infalling matter close to the event horizon, the deviation of stellar orbits from Newton dynamics (relativistic periastron shift and rotation), the precession of the angular momentum vector around the spin of the black hole (Lense-Thirring Precession), redshift variations in the spectrum of orbiting stars, etc. General relativity clearly represents a golden key to unravel the physical features of the black hole (spin, inclination, etc.) and distinguish among the various (competitive) theoretical models (Jaroszynski 1998b; Fragile & Mathews 2000; Rubilar & Eckart 2001; Broderick & Loeb 2005; Weinberg et al. 2005; Zucker et al. 2006; Will 2008; Broderick et al. 2009; Hamaus et al. 2009; Merritt et al. 2009; Merritt 2010; Angélim & Saha 2011).

Another very interesting effect expected from general relativity is *gravitational lensing*. In fact, Sgr A*, acting as a powerful gravitational lens, is able to deflect the light rays emitted by the S-stars from their trajectories, affecting their measured positions and eventually generating secondary images (Wardle & Yusuf-Zadeh 1992; Jaroszynski 1998a; Alexander & Sternberg 1999; Chanamé et al. 2001; De Paolis et al. 2003; Bozza & Mancini 2004; Nusser & Broadhurst 2004; Bozza & Mancini 2005, 2009). However, the expected astrometric displacements of the primary image ($\sim 20 \mu\text{as}$, Gillessen et al. 2009a) and the secondary-image luminosities ($K = 20.8$ in the present known best case, Bozza & Mancini 2009) are difficult to detect and the resolution of the most powerful modern instruments is currently insufficient to perform such high precision astrometry and photometry. However, the perspectives for observing S-star displacements and secondary images generated by the massive black hole at the Galactic center are not so far from what the modern technology is going to put into play. Innovative NIR interferometry instruments are now under development at the Very Large Telescope and Keck: they are PRIMA (Delplancke 2008), GRAVITY (Eisenhauer et al. 2008; Gillessen et al. 2010), and ASTRA (Pott et al. 2008). These

instruments have been conceived to achieve an astrometric accuracy of $10 - 100 \mu\text{as}$ in combination with milli-arcsec angular-resolution imaging.

Here we focus on the new second-generation Very Large Telescope Interferometer (VLTI) instrument, GRAVITY, that will be scientifically operative at the ESO Paranal observational site starting supposedly from 2014 (Eisenhauer et al. 2008). Its precision will allow to study the close environment of Sgr A* in detail, testing general relativity in this so striking laboratory. The aim of the present paper is analyze the ability of GRAVITY to detect gravitational-lensing effects generated by the central black hole, estimating the position and the luminosity of the primary and secondary images of the S-stars as functions of time.

Our paper is structured as follows. In § 2 we review the current NIR observations that have lead to the present knowledge of the Galactic center region. We also introduce GRAVITY and its expected performance. In § 3 we examine the gravitational lensing effects by dividing the space around Sgr A* in three regions according to the GRAVITY ability to separate the images of a stellar source from the black hole. In § 4, 5, and 6 we illustrate the respective results in the three regions. In § 7 we draw the conclusions.

2. NIR observations toward the Galactic center

From a physical point of view, the central region of the Milky Way is one of the most interesting zones of the Universe. Being located at $R_0 = 8.3 \text{ kpc}$ from the Solar system (Gillessen et al. 2009a), it is the nearest and most accessible place where to study the interplay between a central massive black hole with the gas and the stars in its environment. This region is also characterized by a very high interstellar extinction (roughly 30 mag in the optical band) along our line of sight, since it crosses the disk and several spiral arms of the Galaxy. Nevertheless, the Galactic center is quite transparent in the NIR bands, where the extinction amounts to only $\approx 3 \text{ mag}$. In particular, a detailed study performed by Schödel et al. (2007) showed that the interstellar extinction in the central $\approx 0.5 \text{ pc}$ is highly variable and has a general minimum centered on Sgr A* of $A_K = 2.8$ (Eisenhauer et al. 2005).

2.1. Current NIR observations

After the first observations by *speckle imaging*, which have placed the first constraints on the existence of a massive black hole (Eckart et al. 1993; Genzel et al. 1996; Eckart & Genzel 1996, 1997; Ghez et al. 1998), the NIR observations (both imaging and spectroscopy) of the galactic center are currently performed thanks to the adaptive optics and laser guide technique, two very innovative technologies introduced in the large-class telescopes ($8 - 10 \text{ m}$) since 2002 (Schödel et al. 2002; Ghez et al. 2003; Schödel et al. 2003; Eisenhauer et al. 2005; Ghez et al. 2005b, 2008; Gillessen et al. 2009a). Thanks to these instrumentations the imaging observations can go about 2-3 mag deeper

than previous speckle imaging observations (Schödel et al. 2007).

Considering the high density of the star cluster in the neighborhood of Sgr A*, a 8 m-class VLT unit telescope is able to detect a faint star down to an apparent magnitude of 18.5 in the K band (around $\lambda = 2.2 \mu\text{m}$). Instead, the limit of the spectroscopical measurements allows to identify early- and late-type stars as faint as $K \approx 16.5$ and $K \approx 17$, respectively. For the brightest S-stars, it is also possible to achieve astrometric and radial-velocity measurements with a precision in the range $200 - 300 \mu\text{as}$ and 15 km s^{-1} , respectively (Fritz et al. 2010; Gillessen et al. 2011).

2.2. Sgr A*

The center of the Milky Way hosts a very exotic source, which is variable across roughly the entire electromagnetic spectrum. It has been detected for the first time in the radio wavelength many years ago (Balick & Brown 1974), and nowadays we know that this source is very compact and motionless, and its location coincides with the dynamical center of the Galaxy (Reid & Brunthaler 2004). Over the last decade, Sgr A* has been fully analyzed across the electromagnetic spectrum, at radio, sub-millimeter, NIR, and X-ray wavelengths, revealing itself as a highly variable source especially in the X-ray and NIR bands. Very extensive observations in the K band taken at NACO/VLT in 2004 – 2009 showed that Sgr A* has a continuously variable low-level state. The low-level emission above 5 mJy identifies the *flaring emission state* and the low-level emission below 5 mJy is considered as the *quiescent state*, with at least 0.5 mJy of the flux due to a faint stellar component. The quiescent state is variable with four NIR peaks and one X-ray peak a day. Flares that have more than 5 mJy of variability are quite rare, roughly two per year (Dodds-Eden et al. 2011).

2.3. S-Stars

The central 10 arcsec region of the Milky Way hosts a stellar cluster, formed by old, late-type red giants, supergiants, asymptotic giant branch stars, but also by many hot young, early-type stars, like post-main sequence blue supergiants and Wolf-Rayet stars. In particular, deep NIR observations of the Galactic center (both proper-motion and Doppler measurements) revealed ≈ 100 stars within $r = 1''$ ($\approx 0.04 \text{ pc}$) of Sgr A*, with a remarkable concentration of B stars, the so-called S-cluster¹.

The existence of so many young stars in this place raises an intriguing paradox (Ghez et al. 2003). Due to the intense gravitational field generated by the central black hole, it is improbable that they formed in situ, but on the other hand their age ($2 - 8 \text{ Myr}$) is not in agree-

¹Unfortunately, the MPE and the UCLA groups used different nomenclature for the stars belonging to this cluster.

ment with migration time scale. Other alternative scenarios have been postulated, including a mechanism of exchange capture of young binaries and the demand of an intermediate mass black hole (Gould & Quillen 2003; Milosavljević & Loeb 2004; Harfst et al. 2008; Löckmann et al. 2009; Madigan et al. 2009; Perets et al. 2009; Merritt et al. 2009; Griv 2010; Alexander 2011).

By using the above cited high-resolution near-infrared techniques, 16 years of extensive monitoring of stellar orbits around Sgr A*, have allowed to refine the orbital parameters of many S-stars. At the present time, the orbits of 27 S-stars have been well determined (Gillessen et al. 2009a). In a gravitational-lensing context, these stars have already been analyzed by Bozza & Mancini (2005, 2009), who calculated all the properties of their secondary images, including time and magnitude of their luminosity peaks and their angular distances from the central black hole. Currently, the most interesting case came out to be S6, since its secondary image will reach $K = 20.8$ at its peak in 2062, with an angular separation of 0.3 mas from the central black hole.

2.4. GRAVITY

A big step in angular resolution is expected to come from NIR interferometry. Actually, GRAVITY is a second-generation VLTI instrument, specifically designed to observe highly relativistic motions of matter close to the event horizon of Sgr A*. With a baseline of ≈ 100 m, it should ensure the access to its innermost stable circular orbit (the radius of this orbit is $30 \mu\text{as}$ for a $4.3 \times 10^6 M_\odot$ Schwarzschild black hole), allowing astrometric detection of *hotspots* orbiting around the black hole. Thus, the high sensitivity of GRAVITY should allow to reconstruct images with details as faint as $K = 19$, yielding a resolution of ≈ 3 mas for objects that can be as faint as $K = 18$. With this remarkable resolution, GRAVITY should be able to catch the light of most of any very fast orbiting stars within the central 100 mas. Its design will finish during 2012 and it will be presumably installed in Paranal in 2014. (Eisenhauer et al. 2008; Gillessen et al. 2010, 2011).

According to recent numerical simulations by Vincent et al. (2011), in the pure imaging mode, GRAVITY should provide an astrometric precision of the order of, or better than, the Sgr A* Schwarzschild radius, for a source limiting brightness of $K < 15$ and with an integration time of 100 s. However, it must be taken into account that the astrometric precision is also very dependent on the number of sources in the field (Gillessen et al. 2010; Vincent et al. 2011).

3. Gravitational lensing phenomena around Sgr A*

The focus of this paper is on the effects of gravitational lensing by Sgr A* on nearby stars and how these affect their apparent properties such as position and luminosity. We will model Sgr A* as a purely Schwarzschild black hole, arguing on the validity of this hypothesis for the proposed observations all along the illustration of the results.

3.1. Basics of lensing

We assume a distance to Sgr A* of $D_{\text{OL}} = 8.3$ kpc and a mass of $M = 4.3 \times 10^6 M_{\odot}$, following Gillessen et al. (2009a). We will refer to the line connecting the observer to the lens as the optical axis. The source position is then fixed in polar coordinates around the lens by a radial distance D_{LS} , a polar angle γ taken from the optical axis, and an azimuthal angle ϕ . The latter is irrelevant in the lensing discussion, given the assumed spherical symmetry of the lens. γ ranges from 0 (perfect alignment of the source behind the lens) to π (perfect anti-alignment, with the source in front of the lens). In Figure 1 we illustrate the notations in the geometric set-up for gravitational lensing.

The Schwarzschild radius of Sgr A* is

$$R_S = \frac{2GM}{c^2} = 1.27 \times 10^{10} \text{ m} = 4.12 \times 10^{-7} \text{ pc}. \quad (1)$$

The angular Schwarzschild radius is thus

$$\theta_S = R_S/D_{\text{OL}} = 10.2 \mu\text{as}. \quad (2)$$

The motion of photons follows null geodesics around a Schwarzschild black hole. We can thus write an exact lens equation relating the source position (D_{LS}, γ) to the image position θ as seen by the observer (cfr. Figure 1) (Frittelli & Newman 1999; Frittelli et al. 2000; Perlick 2004a,b; Bozza 2008, 2010). To this purpose we define

$$A(r) = 1 - \frac{R_S}{r}, \quad (3)$$

i.e. the g_{tt} component of the Schwarzschild metric or equivalently the inverse of the g_{rr} component. Considering that the impact parameter u is related to the image position by

$$u = \frac{D_{\text{OL}}}{\sqrt{A(D_{\text{OL}})}} \sin \theta \simeq D_{\text{OL}} \theta, \quad (4)$$

and that the closest approach distance of the photon r_m is related to this by

$$u^2 = \frac{r_m^2}{A(r_m)}, \quad (5)$$

the lens equation assumes the form

$$2\pi \mp \gamma = \left[\int_{r_m}^{D_{\text{OL}}} + \int_{r_m}^{D_{\text{LS}}} \right] \frac{u}{r \sqrt{r^2 - u^2 A(r)}} dr, \quad (6)$$

where the upper sign holds for the primary image θ_+ and the lower sign holds for the secondary image θ_- appearing on the opposite side.

The integral can be easily evaluated in terms of elliptic functions as widely reported in the literature (Darwin 1959).

The magnification of a gravitational lensing image can be calculated by taking the ratio of an angular displacement in the observer sky and the corresponding displacement in the source surface orthogonal to the emission direction of the photons. Following Bozza (2010), and expressing the result in a synthetic form, we have

$$\mu = \frac{D_{\text{OS}}^2}{D_{\text{LS}}^2 D_{\text{OL}}^2} \frac{u}{\sin \gamma} \left(\sqrt{1 - u^2/D_{\text{LS}}^2} \frac{du}{d\gamma} + u \frac{du}{dD_{\text{LS}}} \right) \quad (7)$$

where $D_{\text{OS}} = \sqrt{D_{\text{OL}}^2 + D_{\text{LS}}^2 + 2D_{\text{OL}}D_{\text{LS}}\cos\gamma}$ is the distance from the observer to the source. The derivatives $du/d\gamma$ and du/dD_{LS} can be evaluated numerically using the lens equation. Note that for small impact parameters compared to the source distance ($u \ll D_{\text{LS}}$) we recover the magnification formula by Ohanian (1987). In the context of black hole lensing, this is sometimes referred to as thin lens approximation.

Gravitational lensing effects are maximal for $\gamma \simeq 0$ (standard lensing), with a second maximum for the secondary image occurring for $\gamma = \pi$ (retro-lensing). This is reflected by the $\sin \gamma$ in the denominator of the magnification formula.

In the weak field limit, the integral of Eq. (6) simplifies and the lens equation becomes

$$\pm \gamma = \frac{r_m}{D_{\text{OL}}} + \arcsin \frac{r_m}{D_{\text{LS}}} - \frac{R_S}{r_m} - \frac{R_S(2D_{\text{LS}} + r_m) \sqrt{\frac{D_{\text{LS}}^2}{r_m^2} - 1}}{D_{\text{LS}}(D_{\text{LS}} + r_m)}, \quad (8)$$

where we have just saved the lowest order term in r_m/D_{OL} , whereas we allow D_{LS} to become comparable to r_m .

The relation between the minimum distance r_m and the impact parameter u can be written to first order in R_S as

$$r_m = u - \frac{R_S}{2} + o(R_S). \quad (9)$$

In addition to the weak field approximation, the thin lens approximation requires $D_{\text{LS}} \gg r_m, u$. This automatically implies small angles, so that we can set $\gamma = D_{\text{OS}}\beta/D_{\text{LS}}$, where β is the observed angular position of the source if there were no lens (see Figure 1). The small angles weak field lens equation is then obtained

$$\beta = \theta - \frac{\theta_E^2}{\theta}, \quad (10)$$

where

$$\theta_E = \sqrt{\frac{2R_S D_{\text{LS}}}{D_{\text{OL}} D_{\text{OS}}}} \quad (11)$$

is the so-called Einstein angle.

Summing up, the full general relativistic treatment without approximation is given by Eq. (6). The weak field approximation is given by Eq. (8) and the weak field + thin lens approximation is given by Eq. (10). Note that if we want to test the sensitivity of gravitational lensing to Post-Newtonian orders of the black hole metric, we must compare the results of Eq. (8) to Eq. (6). In fact, in many cases the differences between Eq. (6) and Eq. (10) are not imputable to strong field effects but just to the thin lens/small angles approximation.

Nevertheless, it is useful to recall the simple analytical expressions of images and magnification obtained by Eq. (10), since they provide a useful guide to the expected gravitational lensing effects. We have

$$\theta_{\pm} = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right) \quad (12)$$

$$\mu_{\pm} = \frac{1}{2} \left(\frac{\beta^2 + 2\theta_E^2}{\beta \sqrt{\beta^2 + 4\theta_E^2}} \pm 1 \right) \quad (13)$$

The primary image is always outside the Einstein ring with radius θ_E , while the secondary image is always inside the ring. Both images approach the Einstein ring as γ approaches 0, merging into a single degenerate ring image with radius θ_E for $\gamma = 0$.

3.2. Strong field and spin corrections

Sgr A* offers a unique chance to test GR in a strong field regime. It is thus of primary importance to understand to what extent a weak field Schwarzschild treatment based on Eq. (10) is sufficient to explain all the phenomenology within reach of GRAVITY and at which point the full GR treatment of Eq. (6) is really necessary. This will also indicate where to look for new tests of GR based on lensing observations.

3.2.1. Black holes in alternative gravity theories

Most alternative theories of gravity only become effective in the neighborhood of the event horizon, by predicting space-time metrics deviating from Schwarzschild in such an extreme regime. These deviations are presumed to assume sizeable proportions for Sgr A*, which is the closest massive black hole to us. At the same time, gravitational lensing provides one of the simplest ways to test these alternative theories once the metric is known. For this reason, gravitational lensing by Sgr A* is invoked as the natural astrophysical test in these theoretical studies (see e.g. Bozza (2010); Perlick (2004b) and references therein). This class of alternative metrics includes black holes with scalar fields, string theory black holes, braneworld black holes, wormholes, Born-Infeld gravity black hole.

All these metrics predict appreciable effects for the higher order images, i.e. those images generated by photons looping around the black hole once or more times. However, these higher order images require resolutions of the order of μas or better and exceptional alignments to be detected, since they are extremely demagnified. In the context of lensing of S-stars, higher order images were examined by Bozza & Mancini (2004, 2005) for the Schwarzschild metric, where it is shown that they are always fainter than $K = 30$ for all known S-stars. A first direct attempt to find effects of alternative gravity theories in lensing of S-stars has been lead by Bin-Nun (2010, 2011), who considered a tidal Reissner-Nordstrom metric coming from Randall-Sundrum II theories. The perturbations are larger for images forming very close to the black hole, but become very small for secondary images forming far enough from the black hole ($\Delta K \sim 0.01$ for S6).

Summing up, the common feature of all these metrics is that they reproduce the Newtonian limit far from the black hole, while they differ from Schwarzschild in their Post-Newtonian expansions. The position and the magnification of the images can be calculated analytically beyond the weak field limit of Eqs. (12)-(13) for a generic Post-Newtonian metric, by an expansion in powers of the Schwarzschild radius (Ebina et al. 2000; Lewis & Wang 2001; Sereno 2004; Keeton & Petters 2005). The relative correction is of order

$$\frac{\delta\theta_{\pm}}{\theta_{\pm}} \approx \frac{\delta\mu}{\mu} = \frac{\theta_S}{\theta_{\pm}}. \quad (14)$$

It therefore takes effect only for light rays passing very close to the event horizon. This happens for sources very close to the black hole (e.g. on the accretion disk) or for secondary images generated by sources far from the optical axis. In the latter case, the secondary images also become very faint and difficult to distinguish from the lens. For this reason, S-stars do not seem to be quite ideal candidates to study strong field effects. However, we will see that there are regimes in which higher order deflection terms may become marginally accessible. Therefore, for each observable, we will indicate the region in which a source should be in order to yield sizeable Post-Newtonian effects.

3.2.2. Alternatives to the black hole hypothesis

Alternatives to the standard black hole paradigm come from boson (Torres, Capozziello & Lambiase 2000), fermion stars (Viollier, Trautmann & Tupper 1993) and dark matter concentrations (Tsiklauri & Viollier 1998). While these objects are built so as to reproduce the observed spectrum of the black hole accretion disk (Guzmán & Becerril 2009), their extended nature leaves a fundamental signature in gravitational lensing, since the deflection angle no longer grows when the impact parameter is decreased below the physical radius of these stars. However, the restrictions imposed by their construction scheme makes this difference arise only for light rays approaching closer than a few Schwarzschild radii. So, once more a source very close to the central object is needed.

3.2.3. Modified gravity

Among the many attempts to solve the dark matter/dark energy problems by modifying GR, the MOND theory is the most famous (Milgrom 1983), thanks to its recent relativistic version by Bekenstein (2004), called TeVeS. In this case, gravity is modified at small fields, when the gravitational acceleration drops below a new fundamental constant $a_0 \sim 10^{-10} \text{ m/s}^2$. Gravitational lensing by point-like objects in this context was studied by Chiu, Ko, & Tian (2006). It turns out that deviations from Schwarzschild lensing arise for impact parameters comparable or larger than the scale

$$r_0 = \sqrt{GM/a_0} \simeq 77 \text{ pc.} \quad (15)$$

These effects would thus be important for far background bulge stars lensed by Sgr A*, in which case the competing effect of the stellar mass of the inner clusters that would sum up with the black hole lensing would probably make a clear detection of TeVeS lensing very difficult.

In the case of S-stars, whose distance from Sgr A* is below the pc, TeVes corrections are largely negligible (order nas at most in the astrometric shifts).

3.2.4. Black hole rotation

Rotating black holes are described by the Kerr metric, characterized by the spin parameter a , corresponding to the black hole specific angular momentum, ranging from 0 (Schwarzschild) to M (extremal rotation). In Kerr black holes, the point-like caustic of the single lens becomes a small astroidal caustic, shifted from the optical axis by $aR_S/2M$ in the west direction (if we identify north as the direction of the black hole spin projected on the sky) (Serenio & De Luca 2008). This means, that in a first approximation the Kerr black hole works as a Schwarzschild black hole slightly shifted from its position (Asada & Kasai 2000; Asada, Kasai & Yamamoto 2003). If the center of Sgr A* is pinpointed to very high accuracy (below the angular extension of the Schwarzschild radius), then all gravitational lensing effects will work as if the black hole were displaced by $aR_S/2M$. Since many quantities diverge as the inverse of the distance of the source to the optical axis ($D_{\text{LS}}\gamma$ in our notations), even such a small displacement of the caustic may induce dramatic effects. For this reason, we will also indicate the limits of sensitivity to the spin for each observable we are going to examine.

3.2.5. Thin lens approximation break-out

The last possible break-out of the weak deflection lens equation (10) arises when the source is very badly aligned ($\gamma \gg 1$). Then the primary image cannot be calculated using the so-called thin-lens approximation, which in this context consists in sending the integration limits to infinity in Eq. (6). In this case, in fact, D_{LS} becomes comparable to the impact parameter u .

3.2.6. Image distortion

Finally, we also note that it would be extremely improbable to see any kind of image distortion of the stars around Sgr A*. In fact this is possible only for alignment angles γ of the order of the source angular radius, which in the case of S-stars is of the order of ns of arc. For comparison, in Figure 2 we report the positions of S27 and S6 at the best alignment epoch of their orbit. These two stars, which represent the best known cases, at most get at $\gamma \simeq 3^\circ$. Therefore, we will always consider the images of the S-stars as effectively point-like for GRAVITY and thus described by a PSF. For the same reason, the finite size of the Kerr caustic in a rotating black hole does not lead to any observable effects.

3.3. Three regimes where to look for lensing in the Galactic center

Gravitational lensing generates two images of a given source star. These images might be confused or not within the PSF of Sgr A* itself, depending on the angular position of the images compared to GRAVITY resolution limit of 3 mas. We can therefore distinguish three different regimes for gravitational lensing, occurring in three different regions of the plane (γ, D_{LS}) .

- Region I: No images are resolved from Sgr A*. A single blend appears containing both images of the source star along with the radiation from the central black hole.
- Region II: The primary image is resolved separately while the secondary image is still confused with Sgr A*.
- Region III: Both images are distinguished from Sgr A* and can be studied separately.

Note that $|\theta_-| < |\theta_+|$ so that it is not possible to have the secondary image resolved with the primary unresolved.

The boundaries of the three regions can be easily calculated using the lens equation (6) by imposing $\theta_{\pm} = \theta_{thr} \equiv 3$ mas in the case of GRAVITY and solving for γ as a function of D_{LS} . In Figure 2 we can see the shapes of the three regions in the plane (γ, D_{LS}) .

Region I covers the immediate neighborhood of Sgr A*, when the source is too close to the black hole to generate a distinguishable primary image. Using the weak field/thin lens formalism (Eq. 10), an approximate expression for its boundary is

$$\gamma_{I-II} = \frac{D_{\text{OL}} + D_{\text{LS}}}{D_{\text{LS}}} \theta_{thr} - \frac{2R_S}{D_{\text{OL}} \theta_{thr}}. \quad (16)$$

The tip of region I at $\gamma = 0$ extends up to

$$D_{\text{cr}} = \left(\frac{2R_S}{D_{\text{OL}}^2 \theta_{thr}^2} - \frac{1}{D_{\text{OL}}} \right)^{-1} = 0.018 \text{ pc}. \quad (17)$$

Beyond this point the primary image will always be resolved by GRAVITY whatever the alignment angle γ . The weak-deflection analytical expression tracks the exact boundary extremely accurately, as $\theta_{\text{thr}} \simeq 300\theta_S$.

The possibility to observe both images as separate objects from Sgr A* is restricted to sources within a cone starting at the tip of region I. The analytical expression coming from the weak deflection approximation is just the opposite of Eq. (16)

$$\gamma_{II-III} = -\frac{D_{\text{OL}} + D_{\text{LS}}}{D_{\text{LS}}} u_{\text{thr}} + \frac{2R_S}{D_{\text{OL}} u_{\text{thr}}}. \quad (18)$$

For sources at large distances we have $\gamma_{II-III} \rightarrow 0.39^\circ$.

In the next three sections we will closely examine the three regions just defined, pointing to possible interesting observational perspectives with GRAVITY, considering either photometric or astrometric detection channels.

4. Region I

In region I of the parameter space (Figure 2), the primary image of a generic stellar source is within 3 mas from the black hole. Then, it cannot be resolved from the radiation emitted by Sgr A* and forms a single blend along with the secondary image. Obviously, in this situation it is difficult to extract pure gravitational lensing signals on the lensed star, as everything is mixed with the variable flux from Sgr A*.

4.1. Astrometry

Let us consider astrometric variations first. The centroid of the blend composed of Sgr A*, primary image and secondary image of the lensed star can be calculated by evaluating the position of the images by Eq. (6) and their magnification from Eq. (7). Of course, the weight of the two images with respect to Sgr A* depends on the relative fluxes in the K -band. By considering that a magnitude of $K = 0$ corresponds to 667 Jy and that the extinction is $A_K = 2.8$ (Eisenhauer et al. 2005), Sgr A* has an apparent magnitude of $K \approx 17$ in its quiescent state, corresponding to 1.1 mJy. As mentioned before, this flux is highly variable even within the same day. However, for the sake of simplicity, we have taken $K_{\text{SgrA*}} = 17$ for the calculation of the centroid shift. As for the stellar source, typical values of S-stars range from $K = 13$ to $K = 17$. This also means that if gravitational lensing effects take place, the centroid will be typically dominated by the primary image shift, as Sgr A* is relatively faint.

In Figure 3 we show the expected astrometric shift of the centroid in region I due to gravitational lensing effects for a source with $K = 15$. In practice, we are comparing the centroid position of the blend including Sgr A* and the two lensed images, with the centroid position obtained

with Sgr A* and the unlensed source. As expected, the shift is higher for $\gamma \rightarrow 0$, i.e. for better alignments. Considering that the astrometric sensitivity of GRAVITY is of the order of $30 \mu\text{as}$, gravitational lensing effects would be detectable for a large portion of this region up to $\gamma = 30^\circ$. This anticipates the results of the astrometric effects in region II, to be discussed in the next section, where the primary image is resolved and things are much cleaner. In region I, instead, the variability of Sgr A* would add some noise on the astrometric measurements. A reliable extraction of the gravitational lensing astrometric shift would require follow-up observations of the whole transit of the source behind Sgr A*, so that the noise due to Sgr A* can be efficiently subtracted.

Strong field and spin effects on the total centroid shift in region I are tiny, staying below the μas even for sources closer than 10^{-4} pc.

4.2. Photometry

The second channel for the detection of gravitational lensing effects comes from photometry. In this case as well, the variability of Sgr A* adds noise to the flux variations of the primary and secondary images. We have thus fixed a threshold for photometric detection of lensing magnification effects of $\Delta K = 0.1$. The time-scale of lensing variations depends on the radius and the eccentricity of the orbit of the source S-star. For example, the transit of a star on a circular orbit at a distance $D_{\text{LS}} = 0.01$ pc behind the black hole in the region within 3 mas of Sgr A* lasts about 100 days, scaling with $\sqrt{D_{\text{LS}}}$. As this time is much larger than the scale of the variability of Sgr A*, the two phenomena can be disentangled with suitable follow-up observations.

Figure 4 shows region I split in two zones: the left one where the photometric variation due to gravitational lensing is detectable and the right one where the magnification effect is too small. The three curves track this boundary for three values of the source magnitude in K -band (13, 15, or 17). Note that an S-star with $\gamma \gtrsim 10^\circ$ will not induce any perceptible flux variation even if it were more brighter than S2. Based on our knowledge of the orbits of several S-stars, we can say that GRAVITY will always be able to separate the primary image from Sgr A*. This means that at present we do not know S-stars falling in region I. However, GRAVITY will likely discover more stars that are too close to be detected with present facilities. It appears a reasonable guess that some of them will enter the regime described in this section along their orbit. If this is the case, their gravitational lensing effects can be studied along the lines traced in this section.

Interestingly, the photometric magnification of a stellar source provides a first channel for the detection of the spin of the black hole. In Fig. 4 the dotted line bounds the region in which the displacement of the Kerr caustic discussed in Section 3.2 modifies the magnification in such a way that the total observable flux changes by more than $\Delta K = 0.1$. Of course, in order to distinguish the spin signal, we need to know the relative positions of Sgr A* and the source S-star to very high accuracy from previous observations. Contrarily to the spin, Post-Newtonian terms in spherically symmetric black holes do not give rise to detectable effects.

We finally note that the threshold precision of 10% that we have requested here has been chosen by considering the scatter in the data collected with the Naos-Conica (NACO) system mounted at the fourth unit telescope Yepun of the VLT (see over Figure 8 of Gillessen et al. 2009a)². We are quite confident that GRAVITY should be able to achieve a similar precision in measurements of the same type.

5. Region II

In this region, the primary image of the source star is detached from the black hole, whereas the secondary image is not. Obviously, this regime includes the no-lensing limit with $\beta \gg \theta_E$, in which the primary image is basically unlensed and the secondary image becomes negligible. All known S-stars fall in this region, so that GRAVITY will be able to resolve their primary image all along their orbits (in current VLT observations, instead, S2 and S6 used to be unresolved from Sgr A* for some periods).

In this regime, we can look for four different gravitational lensing effects: astrometric shift of the primary image, magnification of the primary image, centroid shift of the blend composed of Sgr A* and the secondary image, magnification effects on the same blend.

5.1. Astrometry on the primary image

The astrometric shift on the primary image due to gravitational lensing by the central black hole has already been studied in detail by Nusser & Broadhurst (2004). These authors could not envisage the breakthrough in astrometric accuracy promised by GRAVITY, so that their conclusions were not very optimistic for the observational perspectives of such an effect in a short term.

For this reason, it is interesting to revisit the expected astrometric shift in the source coordinate space in the light of GRAVITY expectations. Figure 5 shows contour lines for the astrometric shift at several values. Strikingly, many of the known S-stars would have experienced a measurable astrometric shift during their passage behind Sgr A*. Even at alignment angles as large as $\gamma = 35^\circ$ the astrometric shift is of the order of $30 \mu\text{as}$. This encourages to look for astrometric shift effects on known S-stars as soon as GRAVITY becomes fully operational. To this purpose, in Figure 6 we show the position of the S-stars in the (γ, D_{LS}) plane in the years from 2014 to 2024. We see

²The question of how accurate the K values are, is actually difficult, because is not easy to understand how the systematic uncertainties affect the measurements. For instance, the extinction around Sgr A* is not constant spatially but at the contrary, as showed by the accurate extinction map of Schödel et al. (2007), is very patchy. This means that stars that hang around the Galactic center experience higher and lower extinctions, which modulates the measured fluxes. One of the possible explanation of the S2 flux increasing observed in 2002, when the star passed its pericenter, is just that (Gillessen 2009, private communication).

that S17 will have an appreciable shift for a few years, while several S-stars may have a marginally detectable shift. Interesting targets will be S29 and S2, which will make rapid incursions in this plot as they will pass through their periapse.

Figs. 5 and 6 clearly show that the shift is practically independent of the source distance. This statement can actually be supported by a simple analytical derivation. In fact, since light rays of the primary image pass very far from the event horizon, the weak field lens equation (8) gives the same results as the exact lens equation (6) to very high accuracy (the difference at most reaches $5 \mu\text{as}$ for $\gamma \rightarrow 0$). Conversely, at large alignment angles γ , the small angle approximation (10) fails. Starting from Eq. (8), solving perturbatively for u in powers of R_S and taking the limit $D_{\text{OL}} \gg D_{\text{LS}}$, we find

$$\Delta\theta = \frac{R_S \cos^3 \gamma/2}{D_{\text{OL}} \sin \gamma/2}, \quad (19)$$

which is independent of D_{LS} .

Proceeding in a similar way from the small angles thin lens equation (10), we get

$$\Delta\theta \simeq \frac{2R_S}{D_{\text{OL}}\gamma}, \quad (20)$$

which coincides with the small γ expansion of Eq. (19), but gives an error of order 10% at $\gamma = 30^\circ$ already and thus cannot be used for accurate shift calculations on S-stars.

We finally note that this gravitational lensing shift should be taken into account in any accurate orbit determination undertaken from GRAVITY data. This effect will help setting further constraints on the black hole mass, the mass distribution around it and the orbital parameters of S-stars. As anticipated, Post-Newtonian and spin effects for the primary image shift in region II are negligible.

5.2. Astrometry on the secondary image

In region II the secondary image is still blended with the emission from Sgr A*. As the source aligns with the black hole, the secondary image gets brighter and moves away from Sgr A*. Both these effects contribute to shift the centroid of the blend in the direction opposite to the primary image. Obviously, the centroid shift is determined by the balance between the luminosity of Sgr A* and that of the source S-star.

Similarly to what we did for region I, we calculate the centroid shift of this blend. In Figure 7 we show the detection limits of GRAVITY (set to $25 \mu\text{as}$) for three different values of the source magnitude in the K -band. Interestingly, S27 and S6 at their closest alignment would have fallen inside the detection zone if they were more luminous (S6 has $K = 15.4$ and S27 has $K = 15.6$). No other known S-stars generate secondary images prominent enough to be detected by GRAVITY.

Post-Newtonian effects on the secondary image are larger since this image is formed by light rays passing close to the event horizon. However, in the region of interest in Figure 7, the difference between the exact centroid shift and the shift calculated by the weak field formalism stays below $1 \mu\text{as}$. The spin effects reach at most $2 \mu\text{as}$, one order of magnitude below GRAVITY’s sensitivity. In principle, using a very large sample of observations of different stars whose orbit is very well known, it might be possible to estimate the spin and Post-Newtonian terms by a careful statistical analysis. So, this is an interesting possibility opened by a measure that is in principle feasible with GRAVITY, provided that some S-stars pass through this region. Obviously, the lensing signal is confused here with the variability of Sgr A*, which needs to be subtracted with long-period observations.

5.3. Photometry on the primary image

The magnification of the primary image is another possible channel for detecting gravitational lensing. It is inversely proportional to the alignment angle (as long as the source radius is negligible), so that it requires quite accurate alignments. The expected magnification for the known S-stars is very low (we estimated that the best case is $\mu = 1.01$ for S6) and so surely undetectable also by GRAVITY. If we consider a detection threshold of $\Delta K = 0.1$, the candidate sources must be on the left of the dashed line in Figure 8, i.e. within $\gamma = 1^\circ$. It is a small window for this channel, but not impossible to achieve for S-stars to be discovered yet. By crossing astrometric and magnification measurements it is possible to get even more stringent constraints on the lens nature.

5.4. Photometry on the secondary image

The photometry on the secondary image is complicated by the blending with Sgr A*. The threshold at $\Delta K = 0.1$ allows for a relatively wide detection region depending on the source magnitude. In Figure 8 we see its boundary for three values of the magnitude of the S-star. With respect to astrometry, the region extends at larger angles at small D_{LS} , while it becomes thinner at larger D_{LS} . The difference between the exact and weak field values is smaller than in astrometry and is thus not shown in the figure. Also in this case, no known S-stars fall within the detection limits of GRAVITY.

Note that the detection limit for $K = 17$ practically coincides with the limit for the primary image. This is a trivial consequence of the fact that the magnitude of Sgr A* in its quiescent state has been taken to be $K = 17$, i.e. equal to the unlensed source magnitude.

Post-Newtonian and spin contributions to photometric amplification remain small both for the primary and for the secondary image, starting to be significant only for $\gamma < 1'$.

6. Region III

In this region, the two images of the source star appear distinct from Sgr A*. Obviously, this is the most favorable case for a gravitational lensing detailed study. The mass distribution and the distances can be constrained by simultaneous measurements on the primary and the secondary image position and magnification, so that the greatest benefits for a study of the central environment is achieved. Unfortunately, region III is a cone with aperture angle of $\gamma = 0.39^\circ$ only, so that the chances for such a good alignment are quite low.

Furthermore, we still have to discuss the actual detectability of the secondary image, which is always much fainter than the primary. Following our discussion of GRAVITY performance in Section 2.4, we consider a threshold magnitude of $K = 19$ for the secondary to be clearly detected. Depending on the source intrinsic luminosity, in Figure 9 we draw three detection limits corresponding to this threshold. These limits demand an even higher degree of alignment for the secondary image to be effectively visible.

In the weak field/thin lens formalism, these curves can also be calculated analytically by imposing that the magnification of the secondary image (13) is above the threshold $\mu = 10^{-0.4(19-K)}$, where K is the magnitude of the source. We obtain

$$\gamma = \sqrt{\frac{2R_S}{D_{LS}} \frac{(1 + 2\mu) - 2\sqrt{\mu(1 + \mu)}}{\sqrt{\mu(1 + \mu)}}}. \quad (21)$$

Consider that at a given distance D_{LS} , the radius of the detection zone scales as $D_{LS}\gamma \simeq \sqrt{D_{LS}}$, i.e. the detection zone effectively grows with the distance, contrarily to the impression given by Figure 9.

Another interesting aspect is that throughout region III the contribution of Post-Newtonian and spin effects to the astrometric shift is relatively high: 5 μas for the primary image and up to 10 μas for the secondary image. These values are still below GRAVITY sensitivity. However, as already noted before, a large number of observations can be used to study these effects with a statistical approach.

We finally remind the estimates of previous works on the number of bulge stars that should be lensed by Sgr A* at a given time with a threshold of $K = 23$ for the secondary image is around 10 (Jaroszynski 1998a; Alexander & Sternberg 1999; Chanamé et al. 2001). Here we consider a more conservative threshold for GRAVITY, since we are considering its specific use for interferometry.

7. Discussion and conclusions

Thanks to the coherent combination of the four 8-m Unit Telescopes and the four 1.8-m Auxiliary Telescopes, which can be relocated on 30 different stations, the VLTI is the only interferometer

that allows to perform very high-precision photometry, with mas angular resolution, and a total collecting area of 200 m². In particular, the size of the total field of view combining the four 8-m Units is of 2". Since no other interferometer of such class are currently in the design phase, the VLTI will be at the leading-edge still for many years to come. After the VINCI, MIDI (Leinert et al. 2003), AMBER (Petrov et al. 2007) and the visitor instrument PIONIER (Le Bouquin et al. 2011), GRAVITY arises as the second generation VLTI instrument and will perform high precision narrow angle astrometry as well as interferometric imaging in the *K*-band (2.2 μ m).

Taking into account the capabilities of GRAVITY, we have analyzed the possibility to observe gravitational lensing effected generated by the massive black hole, located in the geometrical center of the Milky Way, on the S-stars orbiting around it.

We have divided the parameter space into three regions (Figure 2), depending on the resolution of the primary and the secondary image from the black hole and we have carefully investigated astrometric and photometric channels for the detection of gravitational lensing effects.

Region I For stellar sources in the immediate neighborhood of Sgr A*, both the primary and the secondary images are confused with Sgr A*.

Gravitational lensing is responsible of a shift in the centroid with respect to the unlensed configuration. This shift is measurable by GRAVITY up to an alignment angle of $\gamma \sim 30^\circ$ (cfr. Figure 3, see Figure 1 for the geometrical definition of γ).

Photometric detection of a magnification effect is possible in the inner region $\gamma < 2 \div 5^\circ$ (Figure 4).

These effects have timescales much longer than the daily variability of Sgr A*, which could be subtracted by a dense sampling.

Region II If the S-star is far enough from the black hole, the primary image is resolved by GRAVITY. All known S-stars fall in this region.

In this case, the astrometric shift of the primary is appreciable by GRAVITY for many known S-stars (Figs. 5-6) in a cone with aperture $\gamma \sim 40^\circ$. This shift must be taken into account in the computation of orbital parameters and may help constraining the mass of the black hole and of its environment. The star S17, in particular, will have the highest astrometric shift for several years.

The magnification effect on the primary is more difficult to see and requires $\gamma < 1^\circ$ (Figure 8).

The secondary image is blended with Sgr A*, but is able to yield a measurable centroid shift in the zone between $1^\circ < \gamma < 8^\circ$ depending on the source distance and magnitude (Figure 7).

The magnification effect on the secondary image might be easier to detect (Figure 8), given the relatively low luminosity of Sgr A* in its quiescent state, but it is affected by the problem of the variability of the comntaminant emission, as mentioned before.

Region III In a cone of aperture $\gamma = 0.39^\circ$, the stars will generate two images well separated from Sgr A*. Of course this is the best situation for investigations using gravitational lensing to determine masses and distances in the Sgr A* environment. By imposing that the secondary image is not too faint, we further restrict the zone suitable for such studies (Figure 9).

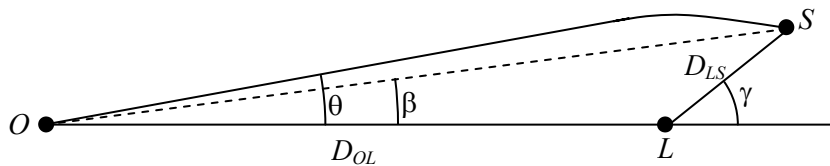
For sources exceptionally aligned on the optical axis and close enough to the black hole, the magnification is sensitive to the spin of the black hole (Figure 4). Apart from this case, spin and Post-Newtonian corrections to the weak field lensing are generally below the sensitivity of GRAVITY in all examined regions. However, the corrections to the astrometric shifts in Region III and the shift of the secondary image in Region II are not far from GRAVITY sensitivity and may be extracted through a careful statistical analysis on a large number of observations.

In conclusion, we can say that GRAVITY will be able to provide the first evidences of gravitational lensing effects by Sgr A*, thanks to its striking astrometric abilities. The astrometric shift due to gravitational lensing must be taken into account in the reconstruction of the orbits of S-stars and can provide independent precious information on the physics of the massive black hole at the center of our Galaxy. New exciting windows on effects beyond the weak field gravity might also be opened if more S-stars are discovered.

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